

Paper Reference 9FM0/3C
Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

Friday 7 June 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

**Diagram Booklet
Answer Booklet**

X75322A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 7 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. [In this question, $\underline{i}$ and $\underline{j}$ are horizontal perpendicular unit vectors.]

A particle **A** has mass 3 kg and a particle **B** has mass 2 kg

The particles are moving on a smooth horizontal plane when they collide directly.

Immediately BEFORE the collision, the velocity of **A** is

$$(3\underline{i} - \underline{j}) \text{ m s}^{-1} \text{ and the velocity of } \mathbf{B} \text{ is } (-6\underline{i} + 2\underline{j}) \text{ m s}^{-1}$$

Immediately AFTER the collision the velocity of **A** is

$$\left(-2\underline{i} + \frac{2}{3}\underline{j}\right) \text{ m s}^{-1}$$

- (a) Find the total kinetic energy of the two particles BEFORE the collision.

(3 marks)

1. continued.

(b) Find, in terms of $\underline{i}$ and $\underline{j}$, the impulse exerted on **A** by **B** in the collision.

(3 marks)

(c) Find, in terms of $\underline{i}$ and $\underline{j}$, the velocity of **B** immediately **AFTER** the collision.

(3 marks)

(Total for Question 1 is 9 marks)

2. A rough plane is inclined to the horizontal at an angle θ , where
- $$\tan \theta = \frac{3}{4}$$

A particle **P** of mass **m** is at rest at a point on the plane.

The particle is projected **up** the plane with speed $\sqrt{2ag}$

The particle moves up a line of greatest slope of the plane and comes to instantaneous rest after moving a distance **d**

The coefficient of friction between **P** and the plane is $\frac{1}{7}$

- (a) Show that the magnitude of the frictional force acting on **P** as it moves up the plane is
- $$\frac{4mg}{35}$$

(3 marks)

2. continued.

Air resistance is assumed to be negligible.

Using the work–energy principle,

(b) find d in terms of a

(4 marks)

(Total for Question 2 is 7 marks)

3. A car of mass **1000 kg** moves in a straight line along a horizontal road at a constant speed of **72 km h^{-1}**

- The resistance to the motion of the car is modelled as a constant force of magnitude **900 newtons**

The engine of the car is working at a constant rate of **P kW**

Using the model,

- (a) find the value of **P**
(3 marks)

(continued on the next page)

3. continued.

The car now travels in a straight line up a road which is inclined to the horizontal at an angle α , where

$$\sin \alpha = \frac{2}{49}$$

- In a refined model, the resistance to the motion of the car from non-gravitational forces is now modelled as a force of magnitude $20v$ newtons, where $v \text{ m s}^{-1}$ is the speed of the car

At the instant when the engine of the car is working at a constant rate of 30 kW and the car is moving up the road at 10 m s^{-1} , the acceleration of the car is $a \text{ m s}^{-2}$

Using the refined model,

- (b) find the value of a
(4 marks)

(continued on the next page)

Turn over

3. continued.

Later on, when the engine of the car is again working at a constant rate of **30 kW**, the car is moving up the road at a constant speed **$U \text{ m s}^{-1}$**

Using the refined model,

(c) find the value of **U**
(5 marks)

(Total for Question 3 is 12 marks)

4. A particle **A** of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. Particle **A** collides directly with a particle **B** of mass m which is at rest on the plane.

The coefficient of restitution between **A** and **B** is e ,
where
 $e > 0$

- (a) Show that the speed of **B** immediately after the collision is
 $2u(1 + e)$
(6 marks)

After the collision, **B** hits a smooth fixed vertical wall which is perpendicular to the direction of motion of **B**

- (b) Show that there will be a second collision between **A** and **B**
(3 marks)

(continued on the next page)

Turn over

4. continued.

The coefficient of restitution between **B** and the wall is

$$\frac{1}{2}$$

Find, in simplified form, in terms of **m**, **u** and **e**,

(c) the magnitude of the impulse received by **B** in its collision with the wall,
(3 marks)

(d) the loss in kinetic energy of **B** due to its collision with the wall.
(3 marks)

(Total for Question 4 is 15 marks)

5. A light elastic string has natural length $2a$ and modulus of elasticity $2mg$

One end of the string is attached to a fixed point **A** on a horizontal ceiling.

The other end is attached to a particle **P** of mass m

The particle **P** hangs in equilibrium at the point **E**, where

$$AE = 3a$$

The particle **P** is then projected vertically downwards from **E** with speed

$$\frac{3}{2}\sqrt{ag}$$

(continued on the next page)

5. continued.

Air resistance is assumed to be negligible.

Find the elastic energy stored in the string, when P first comes to instantaneous rest.

Give your answer in the form $kmga$, where k is a constant to be found.

(Total for Question 5 is 7 marks)

6. [In this question, $\underline{i}$ and $\underline{j}$ are horizontal perpendicular unit vectors.]

A particle **P** is moving with velocity

$(4\underline{i} - \underline{j}) \text{ m s}^{-1}$ on a smooth horizontal plane.

The particle collides with a smooth vertical wall and rebounds with velocity

$(\underline{i} + 3\underline{j}) \text{ m s}^{-1}$

The coefficient of restitution between **P** and the wall is **e**

- (a) Find the value of **e**
(6 marks)

After the collision, **P** goes on to hit a second smooth vertical wall, which is parallel to $\underline{j}$

(continued on the next page)

6. continued.

The coefficient of restitution between **P** and this second wall is

$$\frac{1}{3}$$

The angle through which the direction of motion of **P** has been deflected by its collision with this second wall is α°

(b) Find the value of α , giving your answer to the nearest whole number.

(4 marks)

(Total for Question 6 is 10 marks)

7. Refer to the diagram for Question 7 in the Diagram Booklet.

A smooth uniform sphere **A** of mass **m** is moving with speed **U** on a smooth horizontal plane.

The sphere **A** collides obliquely with a smooth uniform sphere **B** of mass **3m** which is at rest on the plane.

The two spheres have the same radius.

Immediately before the collision, the direction of motion of **A** makes an angle α , where $0^\circ < \alpha < 90^\circ$, with the line joining the centres of the spheres.

Immediately after the collision, the direction of motion of **A** is **perpendicular** to its original direction, as shown in the diagram.

(continued on the next page)

7. continued.

The coefficient of restitution between the spheres is e

(a) Show that the speed of **B** immediately after the collision is

$$\frac{1}{4}(1 + e)U \cos \alpha$$

(6 marks)

(b) Show that

$$e > \frac{1}{3}$$

(4 marks)

(c) Show that

$$0 < \tan \alpha \leq \frac{1}{\sqrt{2}}$$

(5 marks)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
